

Projector Calibration using Arbitrary Planes and Calibrated Camera

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Abstract

In this paper, an easy calibration method for projector is proposed. The calibration handled in this paper is projective relation between 3D space and 2D pattern, and is not correction of trapezoid distortion in projected pattern. In projector-camera systems, especially for 3D measurement, such calibration is the basis of process. The projection from projector can be modeled as inverse projection of the pin-hole camera, which is generally considered as perspective projection. In the existing systems, some special objects or devices are often used to calibrate projector, so that 3D-2D projection map can be measured for typical camera calibration methods. The proposed method utilizes projective geometry between camera and projector, so that it requires only pre-calibrated camera and a plane. It is easy to practice, easy to calculate, and reasonably accurate.

1. Introduction

Projector-camera systems became popular in these years, and one of the popular purposes of them is 3D measurement. We are currently constructing a prototype of 3D measurement system, which requires calibration of projective relation between 3D space and 2D for both of cameras and projectors. The projection from projector can be modeled as inverse projection of the pin-hole camera, which is considered as perspective projection. The only difference between camera and projector is the direction of its projection; 3D scene is projected onto the 2D image-plane in camera, and 2D pattern in projector is projected onto 3D scene. The mathematical theory of the projective geometry is similar in camera and projector.

Then, a straight-forward solution for projector calibration is using camera calibration methods, which generally requires 3D-2D projection maps. In this approach, two complicated operations are required for pre-calibration; *I.* Project a known pattern onto some elaborate object, which has enough variation of depth. *II.* With high precision, measure feature points in the projection pattern on the object.

In this paper, an easy calibration method for projector is proposed. It does not require 3D-2D maps, but requires a pre-calibrated camera. It solves calibration only based on projective geometry between the camera and the projector. The only required object is a plane, and it is easy to practice.

2. Projective Geometry

Generally, 2D-3D projection model of projector is similar to camera, which is perspective projection in ideal condition. The difference between projector model and camera model is just their projective directions. The relation between 2D data and 3D scene is described as perspective projection:

$$\tilde{\mathbf{m}} \propto \mathbf{P}\tilde{\mathbf{M}} \quad (1)$$

Here, $\tilde{\mathbf{M}} = [x, y, z, 1]^T$ is 3D geometry in the scene, and $\tilde{\mathbf{m}} = [u, v, 1]^T$ is 2D geometry in captured image or projection pattern. Such a projection matrix \mathbf{P} is described with intrinsic matrix and extrinsic matrix:

$$\mathbf{P} = \begin{vmatrix} \alpha & 0 & C_x \\ 0 & \beta & C_y \\ 0 & 0 & 1 \end{vmatrix} | \mathbf{R}, \mathbf{t} \quad (2)$$

In intrinsic matrix, α and β mean focal length with skew parameter, and (C_x, C_y) is optical center. In extrinsic matrix, \mathbf{R} and \mathbf{t} are rotation and translation respectively.

Because both of camera and projector are perspective projection, arbitrary plane in the scene provides a homography:

$$\tilde{\mathbf{m}}_a \propto \mathbf{H}_{ba}\tilde{\mathbf{m}}_b \quad (3)$$

2D point $\tilde{\mathbf{m}}_b$ is a 2D geometry in the projector pattern, and so it is projected onto the plane in the scene. Then, that point on the plane is captured in 2D position $\tilde{\mathbf{m}}_a$ of the camera image. Of course, inverse relation \mathbf{H}_{ab} also exists as the inverse matrix of \mathbf{H}_{ba} .

In the same way, projector and camera must have a fundamental matrix, which describes epipolar geometry between projector and camera. Fundamental matrix can be described by two perspective projections and an epipole:

$$\mathbf{F}_{ba} \propto [\tilde{\mathbf{e}}_a]_{\times} \mathbf{P}_a \mathbf{P}_b^+ \quad (4)$$

3. Proposed Method

3.1. Solution of Fundamental Matrix

Fundamental matrix can be solved by several homographies. By moving a plane in the scene, a bunch of homographies can be acquired. Let H_{ba_i} and H_{ab_i} be a pair of homographies observed by a plane i . Then, nameless 3×3 matrix H_{aa} is calculated with two positions of the plane:

$$H_{aa} = H_{ba_i} H_{ab_j} \quad (i \neq j) \quad (5)$$

Using such a matrix H_{aa} , a point \tilde{m} which satisfies

$$\tilde{m} \times H_{aa} \tilde{m} \quad (6)$$

must be a point on the cross-sectional line of plane i and j , or a point projected from the focal point of P_b onto the projection P_a 's image plane. Therefore, epipole \tilde{e}_a can be acquired by Eigen decomposition of H_{aa} [1]. Using the acquired epipole and arbitrary homography, fundamental matrix is calculated as:

$$F_{ba} \propto [\tilde{e}_a]_{\times} H_{ba_i} \quad (7)$$

In the same way, epipole \tilde{e}_b and F_{ab} can be acquired.

3.2. Estimation of Projection Matrix

As noted in section 1, camera calibration is assumed to be known. Let a matrix P_a be the known camera projection, and let P_b be the unknown projection of the projector. From the numerical formula (4) and (7), we can say:

$$\begin{aligned} [\tilde{e}_a]_{\times} H_{ba} P_b &\propto [\tilde{e}_a]_{\times} P_a \\ [\tilde{e}_b]_{\times} H_{ab} P_a &\propto [\tilde{e}_b]_{\times} P_b \end{aligned} \quad (8)$$

Because P_a is known, homographies are acquired, and both epipoles are calculated from homographies, only the target matrix P_b is unknown in these conditions.

In the first step of estimation, we set some reasonable intrinsic matrix. Then extrinsic parameters and focal length α and β are optimized to fit the conditional expressions, using acquired homographies and their solved epipoles. Because the focal points of P_a and P_b are restricted on mutual back-projection lines from epipole e_a and e_b , the extrinsic matrix has only 2 DOF (degrees of freedom) as shown in figure 1. Unfortunately, 1 DOF in translation can not be determined by only this condition, because it corresponds to the scale factor of perspective projection. It can be solved by using multiple cameras or additional information by known sized object. When absolute scale is not required, this 1 DOF of translation is set arbitrarily.

Next, both of the intrinsic and the extrinsic parameters are optimized with the same conditional expressions, using acquired homographies. In this time epipoles are calculated from the evaluating parameters. Therefore, the projector calibration P_b is fully optimized.

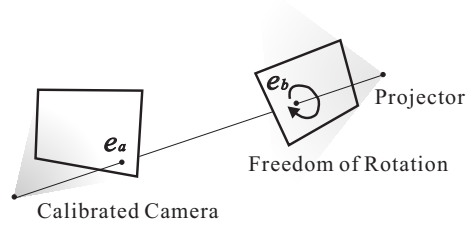


Figure 1. 2 DOF (degrees of freedom) in extrinsic parameters; 1 DOF in rotation is explicitly shown. 1 DOF in translation is the distance of two focal points

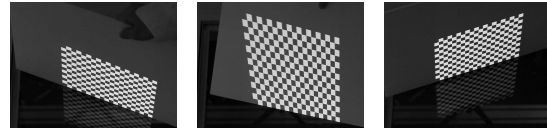


Figure 2. Examples of calibration images for projector

4. Experiments

In our experiments, two XGA (1024×768) cameras and a XGA data projector were used. The camera calibrations' precision was about 0.3 pixel's ambiguity in evaluation points beyond the target scene.

Currently, we are using only QVGA (320×240) sized area in the center of projector pattern. To project full XGA resolution onto the scene, position of the projector must be impractically close to the scene, because our using projector had wide-angled projection. In side effect, using only central part of lens reduces the lens distortion problem.

Eight homographies between the cameras and the projector were acquired based on chessboard pattern as shown in figure 2. Fundamental matrix was acquired by the best pair of the homographies, and all homographies were simultaneously evaluated in the optimization. In evaluation based on points beyond the target scene, the projector's calibration had about 0.4 pixel's ambiguity in cameras. This precision was almost same level of our camera-calibration.

5. Conclusion

An easy calibration method for projector is proposed. The proposed method requires only pre-calibrated camera and a plane, and utilizes projective geometry between the camera and the projector. In spite of easy method, it works with reasonable accuracy, because it evaluates a number of homographies at once.

References

[1] R. Hartley and A. Zisserman. *Multiple View Geometry in Computer Vision*. Cambridge University Press, ISBN 0-521-54051-8, second edition, 2004. 2